SIMPLEST INVENTORY MANAGEMENT MODELS

Saipnazarov Sh.A.[1], Khodjabaeva D.[2] (Republic of Uzbekistan)

[1]Saipnazarov Shaylovbek Aktamovich - Candidate of Pedagogical Sciences, Associate Professor;
[2]Khodjabaeva Dilbar - Senior Lecturer,
DEPARTMENT OF MATHEMATICS,
TASHKENT ECONOMIC UNIVERSITY,
TASHKENT, REPUBLIC OF UZBEKISTAN

Abstract: *this article covers inventory management tasks. We have considered the simplest model, the Wilson model. Too little may not be enough for everyone, and too often you will have to bring new lots, drive vehicles. Hence, it is necessary to find the best value of the stock – not too large and not too small. We mathematically found out that the optimal plan should be sought among those in which the parties arrive at the same time intervals and, therefore, have the same size Q. The plan in which the sizes of all parties are the same and equal to* Q_{opt} *will be called the Wilson plan. Plans Wilson in spite of its simplicity, give usually a large economic effect and because is widely used.*
Keywords: *management, optimal value, warehouse, inventory management theory, model, costs.*

ПРОСТЕЙШИЕ МОДЕЛИ УПРАВЛЕНИЯ ЗАПАСАМИ

Саипназаров Ш.А.[1], Ходжабаева Д.[2] (Республика Узбекистан)

[1]Саипназаров Шайловбек Актамович - кандидат педагогических наук, доцент;
[2]Ходжабаева Дильбар - старший преподаватель,
кафедра математики,
Ташкентский экономический университет,
г. Ташкент, Республика Узбекистан

Аннотация: *задачи управления запасами. Мы рассмотрели простейшую модель - модель Вильсона. Слишком мало может не хватить на всех, и слишком часто вам придется привозить новые партии, гонять транспорт. Следовательно, необходимо найти лучшую величину партии - не слишком большую и не слишком маленькую. Мы математически выяснили, что оптимальный план следует искать среди тех, в которых партии прибывают в одинаковые промежутки времени и, следовательно, имеют одинаковый размер партии. План, в котором размеры всех партий одинаковы, будем назвать планом Вильсона. Планы Вильсона, несмотря на свою простоту, обычно дают большой экономический эффект и потому широко используются.*
Ключевые слова: *управление, оптимальная стоимость, склад, теория управления запасами, модель, затраты.*

Introduction. Mathematics can help plan the operation of factories, and stores. A wide variety of supplies are stored in warehouses and pantries: bricks and perfume, tractors and sugar, books and bread, tires and soda… Too many stocks is bad, materials are wasted, and bread can dry out. Too little may not be enough for everyone, and too often you will have to bring new lots, drive vehicles. Hence, it is necessary to find the best value of the stock – not too large and not too small.

The mathematical theory of inventory management is now developing rapidly. Thousands of books and articles have been printed and a wide variety of models created and used. We will consider the simplest Wilson model, which, despite its simplicity, is widely used and brings great benefits.

Let's move on to describing a real situation, for making a decision in which we build a mathematical model .

Let's denote by the letter "r" daily demand. A change in the stocks "S" product in the warehouse can be represented by a broken line (Fig. 1).

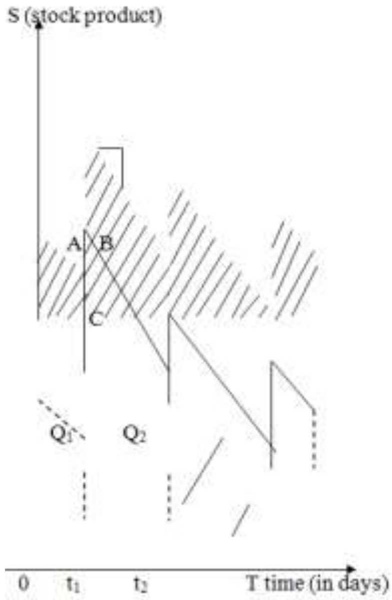

Fig. 1. Change in stock of products in the warehouse

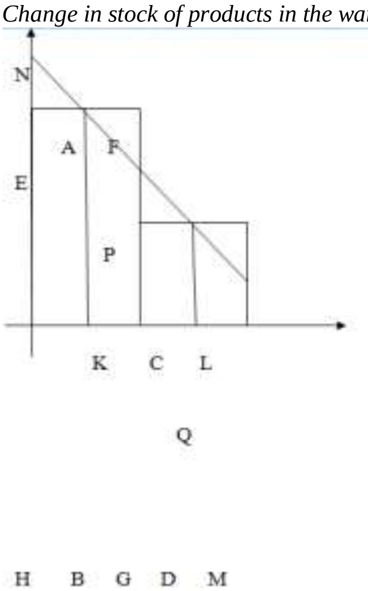

Fig. 2. Product stock change in one day

This means that the cost of maintaining the product for the first day is proportional to S_1, and for the next one is proportional to S_2. Costs for 2 days are proportional S_1+S_2 with the same coefficient of proportionality. And now we will use the fact that areas of triangles NAE and AFP are equal, since they are rectangular, the angles NAE and AFP are equal, as vertical, NA=AP, (because HB=BG by construction, the lines EH, AB, FG are parallel), and the triangles under consideration are equal [1].

Therefore, S_1, is equal to the area of the trapezoid NKGH, and S_2 is equal to the area of the trapezoid PQMG for similar reasons, and S_1+S_2 is equal to the area of the trapezoid NQMH. Thus, we have a convenient expression for calculating the costs for several days: they are proportional to the area under the graph of the amount of stock limited the abscissa axis and vertical straight lines corresponding to the beginning of the first day and the end of the last. During the time T, the products will stay together in the warehouse for as many days as the shaded area under the graph in Figure 1, if we assume that h is a unit of measurement on the abscissa axis, and the time T, the average cost per day will be equal [2].

$$a = \frac{1}{\mathrm{T}}\{\mathrm{G}[\text{Number of deliveries during time T}] + \mathrm{F}[\text{Area under the graph}] \qquad (1)$$

Now we have completely switched to the language of mathematical. It is necessary to solve a purely mathematical problem to minimize the value of (1), having found the optimal supply plan, I, e. Lot sizes $Q_{0,}Q_{1,}Q_2,\ldots$ and delivery times t_1 ,$t_2,\ldots$

We will show that, optimally, all lot sizes are equal and the intervals between their deliveries are also equal.

First, in the set of plans, select a subset that contains the optimal plan, take some plan and try to improve it (Fig 3). It is not profitable to have a stock when the next batch arrives. If the first tooth in Figure 3 (solid line) is replaced by the one shown by the dashed line so that the value of the stock at the moment of arrival of delivery Q_1 is equal to O, then the costs will decrease.

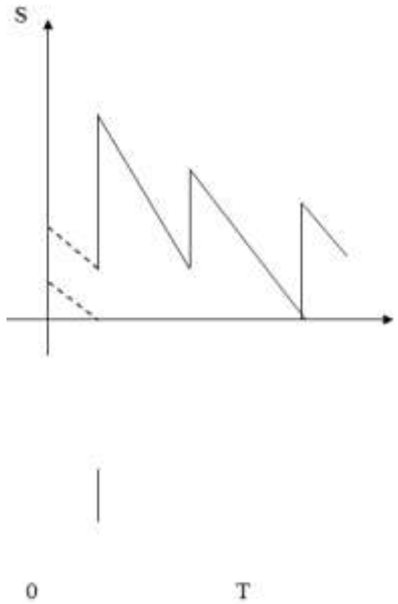

Fig. 3. Moment of arrival of delivery Q1 is equal to O, then the costs will decrease

Indeed, the number of orders and the moments of their delivery will remain the same, and the area under the graph will decrease by the shaded area. You can do the same with the rest of the teeth. This means that the optimal plan must be sought among those in which all teeth reach the T axis [3].

In other words, if the plan has at least one tooth does not reach the ‘’T’’ axis, then this plan is not optimal. If all the waves reach the T- axis, then we can determine the quantities, knowing the moments of their arrival. Indeed, batch Q_1 is consumed from time t_i to time t_{i+1}- the arrival of the next batch. Consequently,

$Q_i = r(t_{i+1} - t_i)$

Here “r” is the daily demand. The cost of maintaining the product from moment t_i to moment t_{i+1} is easy to calculate. Below the graph is a triangle with base $(t_{i+1}-t_i)$ and height $Q_i=r(t_{i+1}-t_i)$. It’s area is equal to:

$$\frac{1}{2}Q_i(t_{i+1} - t_i) = \frac{1}{2}r(t_{i+1} - t_i)^2,$$

and the cost of maintaining the product is “F” times greater and equal

$\frac{1}{2}rF(t_{i+1} - t_i)^2.$

During time “T” it can be 1,2,3,…delivery of products to the warehouse (the first occurs at time 0). As already established, the optimal plan is among those in which all teeth reach the T axis. In particular, the stock at time “T” is 0. Now we find the optimal plan provided that exactly “K” batches are delivered in time “T” at times t_0, t_1, …,t_{k-1} respectively. After that, all that remains is to choose the optimal “K”. Let us introduce the notation:

i=1,…,k-2,

$$\Delta_{i+1} = t_{i+1} - t_i, \Delta_1 = t_1, \Delta_k = T - t_{k-1}$$

Delivery costs are equal to GK and are the same for all considered plans. The costs of maintaining the product from moment t_i to moment t_{i+1} are equal $\frac{1}{2}rF\Delta_i^2$, and for the entire time “T” are equal

$$\frac{1}{2}rF(\Delta_1^2 + \Delta_2^2 + \cdots + \Delta_{k-1}^2 + \Delta_k^2)$$

To minimize (1), it is enough to minimize (2). We get the following math problem; For which non-negative numbers x_1, x_2, i=1, 2,…,k, the sum of which is equal to T, does the value attain a minimum

$$\Delta_1^2 + \Delta_2^2 + \cdots + \Delta_{k-1}^2 + \Delta_k^2 \qquad (2)$$

Consider first the particular case k=2. Let’s introduce the numbers x_1, x_2 using equalities.

$$\Delta_1 = \frac{T}{2} + x_1, \Delta_2 = \frac{T}{2} + x_2.$$

Then

$$x_1 + x_2 = \left(\Delta_1 - \frac{T}{2}\right) + \left(\Delta_2 - \frac{T}{2}\right) = \Delta_1 + \Delta_2 - T = 0.$$

Now let's calculate the sum of squares;

$$\Delta_1^2 + \Delta_2^2 = \left(\frac{T}{2} + x_1\right)^2 + \left(\frac{T}{2} + x_2\right)^2 = \frac{T^2}{4} + Tx_1 + x_1^2 + \frac{T^2}{4} + Tx_2 + x_2^2 = \frac{T^2}{2} + T(x_1 + x_2) + x_1^2 + x_2^2 = \frac{T^2}{2} + x_1^2 + x_2^2.$$

Therefore $\Delta_1^2 + \Delta_2^2$ reaches a minimum when it reaches a minimum of $x_1^2 + x_2^2$. And the sum of squares $x_1^2 + x_2^2$ is always non negative and is equal to 0 only in the case $x_1=x_2=0$. Hence, in the case under consideration, (2) reaches a minimum when $\Delta_1 = \Delta_2 = \frac{T}{2}$.

Now let us prove in the general case.

$$\Delta_i = \frac{T}{k} + x_i \, , i = 1, \ldots, k.$$

Then

$$x_1 + x_2 + \cdots + x_k = \left(\Delta_1 - \frac{T}{k}\right) + \cdots + \left(\Delta_k - \frac{T}{k}\right) = \Delta_1 + \ldots + \Delta_k - T = 0.$$

Let's calculate the sum of squares:

$$\Delta_1^2 + \cdots + \Delta_k^2 = \left(\frac{T}{k} + x_1\right)^2 + \cdots + \left(\frac{T}{k} + x_k\right)^2 == \left(\frac{T^2}{k^2} + 2\frac{T}{k}x_1 + x_1^2\right) + \cdots + \left(+\frac{T^2}{k^2} + 2\frac{T}{k}x_k + x_k^2\right) =$$

$$= \frac{T^2}{k} + \frac{2T}{k}(x_1 + x_2 + \cdots + x_k) + (x_1^2 + x_2^2 + \cdots + x_k^2) = \frac{T^2}{k} + x_1^2 + x_2^2 + \cdots + x_k^2.$$

The sum of squares $x_1^2 + x_2^2 + \cdots + x_k^2$ is always non-negative and is equal to 0 only in case $x_1 = \cdots = x_k = 0$. Hence , the sum of squares $\Delta_1^2 + \cdots + \Delta_k^2$ reaches its minimum in case

$$\Delta_1 = \cdots = \Delta_2 = \frac{T}{k}.$$

We found out that the optimal plan should be sought among those in which the parties arrive at the same time intervals and, therefore, have the same size "Q" (Fig. 4). The gap Δ between the arrival of parties is expressed in terms of "Q" as follows: $\Delta = \frac{Q}{r}$.

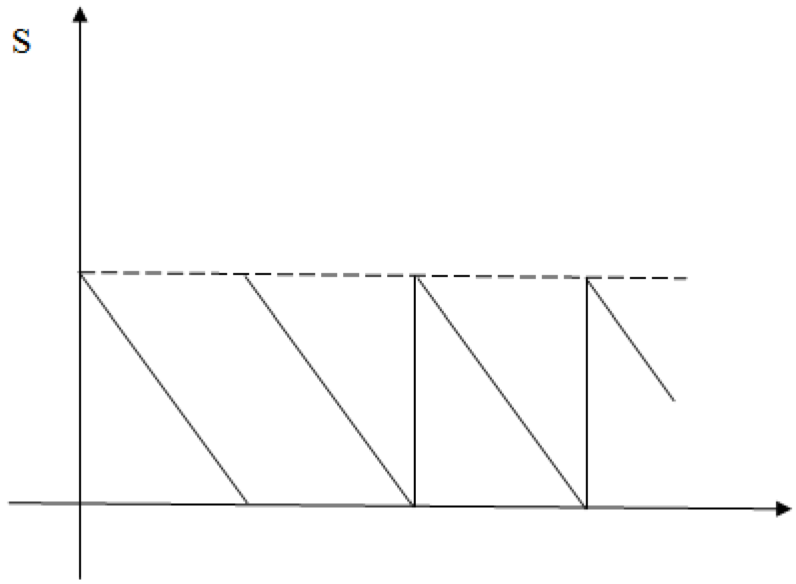

Fig. 4. The gap Δ between the arrival of parties is expressed in terms of "Q" as follows: Δ=Q/r

Let ℓ batches of "Q" products completely diverge in time $T = \ell\Delta$. Let's calculate the average cost (1) for a plan in which all batches are of size Q. The total demand in time T is rT. During this time , $\ell = \frac{rT}{Q}$ batches of products are delivered. Below the graph of $\frac{rT}{Q}$ triangles, the area of each is $\frac{Q^2}{2r}$, and the total area under the graph is as follows: $\frac{Q^2}{2r} \cdot \frac{rT}{Q} = \frac{QT}{2}$.

For the delivery and maintenance of products in time "T" spent

$$G\ell + \frac{FQT}{2} = G\,\frac{rT}{Q} + F\frac{QT}{2} = \left(\frac{Gr}{Q} + \frac{FQ}{2}\right) \cdot T$$

and the average cost of (1) for one day is equal to:

$$f(Q) = \frac{Gr}{Q} + \frac{FQ}{2} \qquad (3)$$

Now we are going to do the math again:

At what value of "Q" does the function f (Q) reach its minimum? Which Q is the most profitable? From (3) we get

$f'(Q) = -\frac{Gr}{Q^2} + \frac{F}{2} = 0,$ from here

$$Q_{opt} = \sqrt{\frac{2Gr}{F}} \qquad (4)$$

Formula (4) of the most advantageous batch size is named after the American scientist R. Wilson and was obtained in the first quarter of our century.

The plan in which the sizes of all parties are the same and equal to Q_{opt} will be called the Wilson plan. Plans Wilson in spite of its simplicity, give usually a large economic effect and because is widely used.

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